

# Relativistic Rotation in the Large Radius, Small Angular Velocity Limit

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## Abstract

Relativistic rotation is considered in the limit of angular velocity approaching zero and radial distance approaching infinity, such that centrifugal acceleration is immeasurably small while tangent velocity remains close to the speed of light. For this case, the predictions of the traditional approach to relativistic rotation using local co-moving Lorentz frames are compared and contrasted with those of the differential geometry based non-time-orthogonal analysis approach. Different predictions by the two approaches imply that only the non-time-orthogonal approach is valid.

## 1 Introduction

An approach to analyzing relativistic rotation that has been popular since the early 1900s entails use of local Lorentz frames located throughout a rotating frame[1]·[2]·[3]·[4]·[5]·[6]. Each local Lorentz frame has an instantaneous velocity with respect to the center of rotation equal to the rotating frame tangent velocity  $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$  at the point within the rotating frame where the Lorentz frame is located. The desired analysis is done locally using special relativity and then integrated over successive local Lorentz frames to obtain global values which one would presumably measure in experiment. We shall refer to this method as the “traditional approach” to relativistic rotation.

Proponents of this approach generally consider the rim of a rotating disk to undergo Lorentz contraction[7]·[8]. This conclusion is based on the well-known velocity dependent Lorentz contraction that occurs in each local frame, followed by a spatial integration from local frame to local frame all around the disk rim.

Proponents of the traditional approach often cite, as support for the approach, the case of very large radius  $r$ , vanishingly small  $\omega$ , where  $v = \omega r$  is on the order of  $c$ , the speed of light, and centrifugal acceleration  $a = \omega^2 r$  is too small to be measured in any experiment. We refer to this case herein as the “limit case”. In such case, it is reasoned, the local region at radius  $r$  can not be discerned from a true Lorentz frame, since neither  $\omega$  nor  $\omega^2 r$  are different enough from zero to be detectable. Thus, an observer on the rim of such a large, slowly rotating disk, and a second observer adjacent the first but stationary in the non-rotating “lab”, can be considered fixed in separate Lorentz frames. It follows that traditional special relativity theory may then be applied directly in all regards.

An alternative method for analysis of rotation comprises use of differential geometry[9]·[10] and suitable transformations between the rotating and lab frames. As this leads to a rotating frame metric with off diagonal terms, and hence a non-orthogonality between space and time, we refer to this method as the non-time-orthogonal (NTO) approach. The predictions of the traditional and NTO approaches agree in many, but not quite all, respects[11]·[12]·[13]·[14]·[15]·[16].

In this article we compare and contrast the predictions of the traditional and NTO approaches specifically in the limit case, and primarily with regard to Lorentz contraction. Prior to doing so, we make several observations for the non-limit case where acceleration and angular velocity are readily measurable.

## 2 The Traditional Approach

Consider a rotating disk first in the non-limit case wherein  $\omega$  and  $\omega^2 r$  are significant and can be readily measured. According to the traditional approach, an observer in the lab sees rods aligned with, and fixed to, the rim of the disk Lorentz contract. It is then concluded that measurement of the circumference  $C$  and the radius  $r$  will result in  $C \neq 2\pi r$  and therefore the 2D surface of the rotating disk must be curved.

There are two problems with this conclusion. First, according to special relativity, an observer in one frame sees rods in the frame of a second observer (with relative velocity difference from the first) as contracted, but the second observer sees no such contraction of her own rods. Thus, the lab observer may see the disk rim rods as contracted, but an observer on the rim would notice no such contraction. If the disk observer sees no contraction, then the Riemann curvature she measures for her disk surface must be zero, and the surface must be flat[17].

Second, again according to special relativity, each observer sees the other's rods as contracted. Hence, by the traditional logic, the disk observer must see the lab rods as contracted, and therefore conclude that the lab surface is curved. But those of us living in the lab know that this is simply not true, and the analysis appears inconsistent.

In response to these points, I have heard the argument that special relativity does not apply to the disk observer, as her frame is not inertial. My response to this is several fold.

First, I ask for mathematical analysis (real physics, not mere words) showing how Lorentz contraction then occurs on the disk rim in an absolute way, i.e., so that both the disk and lab observers agree that the disk meter sticks are shorter than the lab meter sticks. No one has yet provided this.

Second, the fundamental assumption in using local Lorentz frames is that locally a non-inertial (curved) frame can be represented by an inertial (flat) frame. How can we assume local inertial frames can be used as surrogates at the beginning of the analysis to draw conclusions, and then when inconsistencies arise at the end, simply say that they can not[18]?

Third, in the traditional analysis limit case, the disk observer is considered effectively inertial and all of special relativity applies directly. Thus, she should not see her own meter sticks as contracted, thereby meaning that her disk surface is flat. Yet, nothing in passing to the limit case implied anything other than  $C$  remaining  $\neq 2\pi r$  in that limit.

Fourth, in the limit case the lab meter sticks have to appear contracted to the disk observer, since said observer is now Lorentzian. This means, according to the traditional approach logic, that the lab surface is curved. There is no "out" in this case of claiming the disk is not really inertial, such that things are somehow different from standard special relativity.

Finally, if the contraction for rotation is argued to be absolute in the non-limit case, then it must also be so in the limit case. Yet in the limit case, both observers are considered to be Lorentzian, and hence the Lorentz contraction must be relative (each must see the other's rods as contracted) and can not be absolute.

The traditional analysis thus appears to possess serious internal contradictions.

## 3 Non-time-orthogonal Approach

### 3.1 Background

As noted earlier, analysis of rotating frames using differential geometry and the most widely accepted transformation between the lab and the rotating frame leads to a rotating frame metric with off diagonal time-space components. Thus, in such a frame time is not orthogonal to space (NTO).

As noted in endnote [11] the NTO (or Langevin) metric correctly predicts GPS measured results for the rotating earth frame, whereas the traditional approach does not. Further, as noted in ref. [10], while the NTO approach predicts the traditional time dilation and mass energy increase measured in many cyclotron (i.e., rotation) experiments, it predicts no Lorentz contraction in rotating frames.

### 3.2 NTO Non-limit Case

If, as predicted by NTO theory, there is no Lorentz contraction for rotating frames, in either an absolute or relative sense, then the non-limit case objections to the traditional approach of Section 2 vanish.

In NTO theory, by testing one can distinguish between a velocity associated with a rotating frame (e.g., the velocity of a point on the rim of a rotating disk) and a velocity associated with an inertial frame. A Foucault

pendulum, or any number of means, detects angular velocity  $\omega$ . A spring mass system measures  $\omega^2 r$ , and from it the distance  $r$  to the center of rotation can be determined. The tangential velocity of any point within the rotating frame is thus  $v = \omega r$ , and this can be determined in an absolute sense from within the rotating frame without looking outside. No such absolute determination of velocity is possible for a translating frame. (Note that for simplicity we illustrate this and related points herein using non-relativistic mechanics. Analogous relativistic analyses lead to the same conclusions.[19])

This ability to distinguish between velocities arising from rotation and translation lies at the foundation of NTO theory (see ref. [10].) In fact, the degree of non-time-orthogonality (the slope of the time axis with respect to the circumferential space direction axis) is directly related to  $v = \omega r$ .

## 4 Challenges to the NTO Approach

I have been challenged in private communications to defend NTO analysis in the following ways.

1) Consider a meter stick bolted to, and aligned with the circumference of, a rotating disk. An observer fixed in the lab adjacent the moving rim sees no Lorentz contraction of the disk meter stick (according to NTO theory.) Suddenly the bolt breaks and the meter stick is released. As the meter stick then instantaneously changes from an NTO to a Lorentz frame, it should appear to the lab observer to contract instantaneously. This, it is argued, is unnatural and even impossible. There is no change in velocity, yet there is significant change in meter stick length.

2) Consider the limit case of infinitesimally small  $\omega$  and infinitesimally small  $\omega^2 r$ . Via NTO analysis there is no Lorentz contraction even though  $v = \omega r$  is, say,  $c/2$ . If there is no possible means to detect angular velocity or centrifugal acceleration, then how can one distinguish between a local Lorentz frame with speed  $c/2$  relative to the lab and the presumed NTO frame of the disk? That is, how can a meter stick in the former frame look contracted, whereas one in the latter does not, if one can measure no other difference between the two frames.

3) Consider combining 1) and 2) above. In the limit case, the meter stick attached to the disk is suddenly released. To the lab observer it instantaneously contracts. Apart from the instantaneous nature of the contraction [as already questioned in 1)], there is no way to distinguish between the pre and post release states of the meter stick apart from the contraction. That is, for no detectable change in  $\omega$ ,  $\omega^2 r$ , or apparently any other quantity, there is a very significant change in length. Stated somewhat differently, for a nearly infinitesimally small change in  $\omega$  and  $\omega^2 r$  (from immeasurably close to zero to zero), there is a finite change in observed length.

## 5 Replies to the NTO Challenges

### 5.1 Preliminary Comments

Of course, given enough time, or a large enough spatial region, one can always discern whether one's own frame (or another frame) is rotating or not. There are many ways to do this, e.g., a Foucault pendulum, a measurement of Coriolis type motion of free particles, a Sagnac experiment, etc. In particular, one could compare a meter stick pinned to one's frame with a free fall meter stick. The distance between the two, and the angle formed between the two, will change over time, albeit extremely slowly in the limit case. From these changes one can calculate the value for  $v = \omega r$ , and thence the expected NTO effects and their variation from standard relativistic effects. (As shown in reference [10] the degree of non-time-orthogonality is directly related to  $\omega r$ .)

Additionally, nature always knows the values for  $\omega$  and  $\omega^2 r$  (and hence  $v = \omega r$ .) Even if they are too subtle for us to measure over practical time and spatial intervals, the physical world effect should still be there, and it should agree with our theoretical calculations (given that the theory is correct.) By analogy, consider a radial length in a cylindrical coordinate system that is light years in length. A near infinitesimally small change in angle of the radial line near the origin results instantaneously in a very large displacement circumferentially at the other end. Immeasurably small changes in a system can indeed cause other finite, and readily measurable changes.

However, proponents of the traditional theory advocate use of local Lorentz frames, whose validity for analyzing non-inertial cases has in the past been considered beyond repute. Such Lorentz frames are localized in both time and space, i.e., they have infinitesimal extent in four dimensions. Hence, it is argued, they should be equivalent to the non-inertial frame in question for periods of time and space over which rotation can not be discerned.

I argue that locally time is not orthogonal to space and the effective metric to use locally (which would yield measured values for time and space, i.e., values measured with standard meter sticks and clocks) is

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & b & 0 \\ 0 & 1 & 0 & 0 \\ b & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (1)$$

The off diagonal quantity  $b$  is a function of  $\omega r$ . When  $\omega = 0$ , then  $b = 0$ , and the metric reduces to the Lorentz/Minkowski metric  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ . Metric (1) is the general case local metric. The Minkowski metric, representing a time orthogonal Lorentz frame, is a special case (albeit by far the most common case.) Globally, we can measure  $\omega$  readily, just as we can measure average velocity of an object than travels at constant velocity. In the latter case, the local velocity over infinitesimal time and distance scales (which we can not measure) equals the global average velocity (which we can measure). Similarly, the globally measured  $\omega$  and the concomitant global non-time-orthogonality are mirrored locally, though we are unable to measure them locally.

Regardless of the relevance one may attribute to the above remarks, I will respond to the challenges of section 4 with arguments that are not dependent on them.

## 5.2 Additional NTO Analysis Background

In Section 5.3 of reference [10] I note there is another way to measure  $\omega r$ . If one is on the rotating disk, one experiences a potential energy that equals the integral of the force felt ( $m\omega^2 r$ ) over the radial distance from the rim to the center. One feels oneself inside an effective gravitational field. In classical theory the potential energy is simply  $-\frac{1}{2}m\omega^2 r^2$ . In general relativity total energy of a particle at rest in a potential field is

$$e = mc^2 \sqrt{1 + \frac{2\Phi}{c^2}}, \quad (2)$$

where  $\Phi$  is the classical (Newtonian) potential (potential energy per unit mass). Thus one finds[20]

$$\begin{aligned} e &= mc^2 \sqrt{1 - \frac{\omega^2 r^2}{c^2}} = mc^2 - \frac{1}{2}m\omega^2 r^2 - (\text{higher order terms}) \\ &= mc^2 + V_{\text{classical}} + \dots = mc^2 + V_{\text{relativistic}} \end{aligned} \quad (3)$$

where  $V_{\text{classical}} = m\Phi$ . The total energy is thus the familiar rest energy  $mc^2$  plus the relativistic potential energy.

In effect then, the mass of a particle such as an electron is altered by the potential energy of the field. At the center of rotation (where the potential is zero) this mass is simply the usual one we are familiar with. Deep in a strong potential well, however, it will be markedly less. (Note that kinetic energy is positive and increases the mass, whereas potential energy is negative here and decreases the mass.) The mass change therefore will be

$$\Delta m = \frac{e}{c^2} - m \cong -\frac{1}{2}m\frac{\omega^2 r^2}{c^2}. \quad (4)$$

For  $\omega r$  on the order of  $c$ , this is a significant, finite, and readily measurable change, even when  $\omega$  and  $\omega^2 r$  are infinitesimally small. We just measure the mass of an electron in an experiment and it will tell us the potential. That, in turn, tells us the circumferential speed of our rotating frame, and the degree of non-time-orthogonality.

This parallels the Schwarzschild field situation with its inherent time dilation and meter stick contraction, which are not dependent on the gravitational force felt, but on the potential. One could have a gravitational system with no detectable gravitational force, but significant gravitational potential. Hence, the effect on meter sticks and clocks would be significant and finite, even for an observer who could not discern any gravitational force field.[21]

## 5.3 NTO Answers to the Challenges

The numbers below correspond to those of the challenges listed in section 4.

1) My first reply to the non-limit case where a meter stick is suddenly released is that this is little different from the sudden change in stress in an object attached to a centrifuge that is suddenly released. Stress goes from being extremely high to zero in a theoretically infinitesimal time interval. (Actually, in the real world the time interval is more on the order of perhaps microseconds or so.) Why should Lorentz contraction be any different?

My second reply is that a similar situation occurs in the traditional approach. From earlier arguments in section 2, it seems inescapable that the only feasible way Lorentz contraction could occur on a rotating disk is if it were absolute. That is, both lab and disk observers agree the disk meter sticks are shorter than the lab meter sticks. Then consider what the disk observer sees upon release. The lab meter sticks must look longer to him than his meter sticks when he is attached to the disk. But upon release, special relativity applies, and the lab meter sticks must then look shorter to him. This change from longer to shorter must, by the same logic, be instantaneous, and the traditional approach is therefore subject to identical presumed shortcomings as the NTO approach was considered to have.

My third reply to this challenge is that the transformation from a non-contracted meter stick (as in NTO theory) to a contracted one upon sudden release is not actually instantaneous. As explained in the Appendix, the difference in simultaneity between frames results in a gradual (relative to the speed of light) contraction of the meter stick beginning from the leading edge and propagating toward the trailing edge.

2) Regarding the limit case and the lack of ability to discern  $\omega$  or  $\omega^2 r$  because they are so small, I refer to the argument above in section 5.2. One can still measure potential, hence  $\omega r$ , and therefore the degree of non-time-orthogonality. One then knows the frame in question is an NTO frame by measuring finite quantities. If the lab observer can look at the disk meter stick as it passes by, he can also look at a scale with an election on it (or Avagadro's number of some known type of atom) as it passes by. No inconsistency.

3) To answer the combination of challenges 1) and 2) we merely combine the answers to them above. We distinguish between pre and post release frames for the meter stick via measurements of the mass of a known particle type. That change is not infinitesimal and hence neither will be the rapid change as seen in the lab from uncontracted to Lorentz contracted state for the meter stick.

## 6 Other Considerations

In my discussions about NTO analysis with other physicists certain other, related, issues have been raised that I address below.

### 6.1 Instantaneous Center of Rotation

An object moving with variable velocity along a non-geodesic may twist and turn to various degrees along its path. At any given instant, however, one could measure its instantaneous values of  $\omega$  and acceleration  $\mathbf{a}$ . Taking  $a = \omega^2 r$ , one can then calculate an instantaneous value for  $\mathbf{r}$ , the radial vector from the instantaneous center of rotation.

Note this value has physical significance in the sense Einstein considered so important in his gedanken elevator experiments. What one can measure solely inside one's frame, without looking outside, is central to the entire theory of relativity. This is the basis of the equivalence principle in which acceleration and gravitational force, being indistinguishable (locally), are equivalent.

Hence, from what we can measure inside the frame of our particle with variable velocity, we actually *are* rotating about a center of rotation  $r$  distance away[22]. Of course, if acceleration  $\mathbf{a}$  changes an instant later, then so does  $\mathbf{r}$ . In general, we can imagine  $\mathbf{r}$  (and  $\omega$ ) continually "jitterbugging" all over the place.

In our limit case example, upon release, the  $\mathbf{r}$  vector for the meter stick would suddenly jump from extremely large to zero. One assumes here that the meter stick continues to rotate at  $\omega$  after release.

More generally, an object undergoing general motion can be analyzed using NTO theory by determining the instantaneous center of rotation at each point in time during its trajectory, and applying the methodology of the theory at that instant. This then is a local theory, whose values may be integrated to determine globally measurable quantities.

Consider further, an object moving through space which in actuality would be continually impacting tiny dust and other type particles. Each impact would accelerate it and change its angular velocity. Hence, we could imagine a "lab" observer watching this object. What would she see? Would the object look Lorentz contracted? Would it suddenly switch back and forth from Lorentz contracted to (according to NTO theory) not contracted? Would it never look contracted since to some (minute) degree it is always rotating and accelerating in some manner?

Note that in order for the velocity  $\mathbf{v}$  we see in the lab for the object to be purely due to rotation the acceleration and angular velocity of the object must yield precisely  $\mathbf{v}$ . To see the impact of this, consider a specific example, where an object is moving by us at  $c/3$  and has

$$\omega = 10^{-20} \text{ rad/sec} \quad \omega^2 r = 10^{-12} \text{ m/sec}^2 \quad (5)$$

associated with it. Then

$$r = \frac{10^{-12}}{10^{-40}} = 10^{28} \text{ m} \quad \rightarrow \quad \omega r = 10^8 \text{ m/sec} = \frac{c}{3}, \quad (6)$$

and 100% of the velocity we see for the object is due to rotation.

If the acceleration has another value that is still immeasurable, say one tenth of that above, then

$$r = \frac{\omega^2 r}{\omega^2} = \frac{10^{-13}}{10^{-40}} = 10^{27} \text{ m} \quad \rightarrow \quad \omega r = 10^7 \text{ m/sec} = \frac{c}{30} \quad (7)$$

and only 10% of the speed  $c/3$  that we see for the object is due to its instantaneous rotation. Hence, we would see a Lorentz contraction as if the object had a translational speed of 90% of the speed we see with respect to us.

The point is, that for all practical purposes the angular velocity and acceleration values, when very small, will virtually never combine to produce a rotational tangent velocity anywhere close to the velocity we see for the object passing by us. The velocity we see of any object moving through space where its rotation rate and acceleration are negligible can therefore be considered effectively due entirely to translation and all of the usual (time orthogonal) theory of relativity applies.

## 6.2 Meter Stick Lengths in Three Frames

There is often confusion regarding the lengths of meter sticks seen by three different relevant observers: 1) the lab observer, 2) the rotating disk observer fixed to the rim, and 3) an inertial observer instantaneously co-moving with the disk rim. Each is carrying his/her own meter stick and each is looking at the meter sticks of the two others. What does each see?

We answer this question employing the graphical construction used in the Appendix. As noted there, this is well known means for illustrating the Lorentz contraction resulting from the Lorentz transformation. Here we extend it in Figure 1 below to NTO frames as well. For a derivation of this from the appropriate transformations, see Born [23] and reference [10].

We use the notation of ref [10], where upper case represents inertial frames and lower case the rotating frame. K is the lab frame;  $K_1$  that of an inertial observer (in a jet plane with engine off for illustration) instantaneously at rest with respect to a point on the disk rim; and k that of the observer on the disk rim point.

Note that the ends of each meter stick trace out world lines in spacetime. A given meter stick is considered “seen” by a particular observer when the endpoint events of that meter stick are simultaneous (occur at the same time) for the particular observer. Time intervals are considered infinitesimal such that the disk meter stick moves in essentially the same direction as the jet meter stick for the interval considered. From inspection of Figure 1, one can glean the significance with regard to Lorentz contraction of the NTO nature of the rotating frame.

The message of Figure 1 is summarized in Table 1 below. The left hand column of the table represents the observer. Each successive column represents the respective meter stick at rest in the given frame (top box). The comment in each box describes how the observer in her frame sees the meter stick length in the other given frame compared to the meter stick she is carrying with her in her own frame.

Observer below sees meter sticks at right as	K (lab)	$K_1$ (jet)	k (disk)
K (lab)	–	contracted	same
$K_1$ (jet)	contracted	–	longer
k (disk)	same	contracted	–

Table 1: Length of Meter Sticks in Other Frames  
Seen by Given Observers

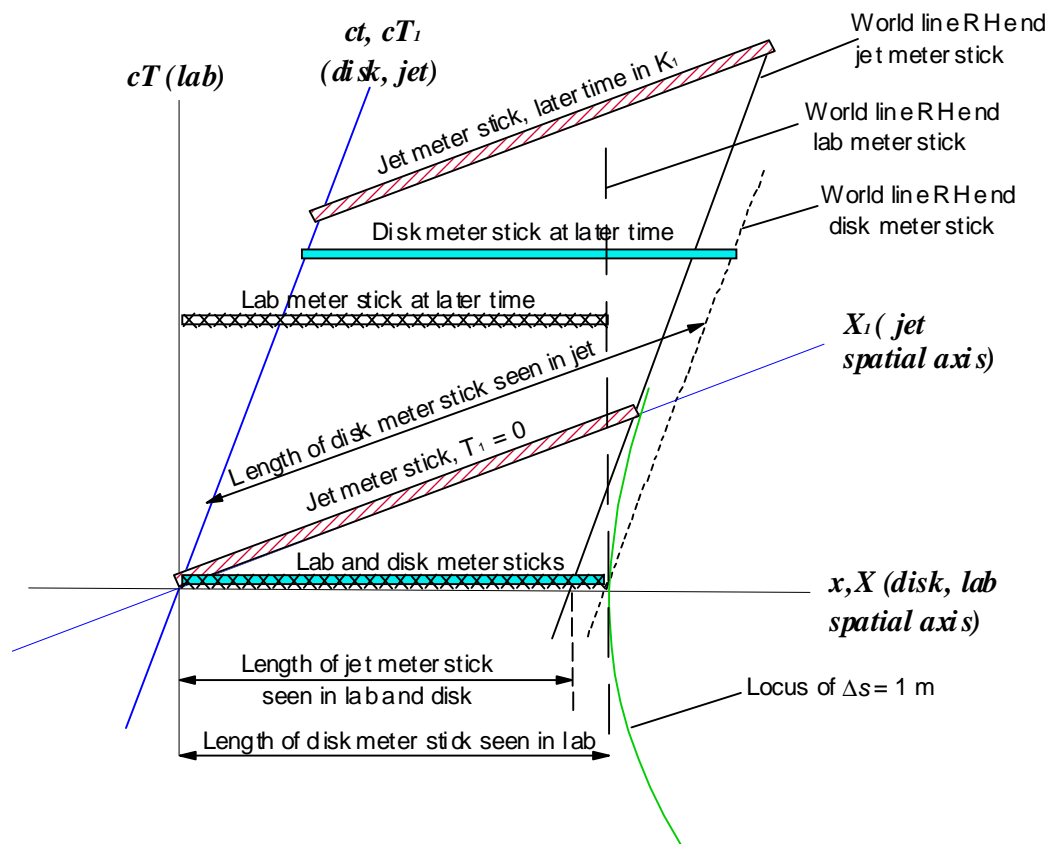


Figure 1: . Length of Meter Sticks in Other Frames Seen by Given Observers

Also, by comparison of columns above, we see the following.

The lab observer in K sees the  $K_1$  (jet) meter stick as shorter than the k (disk) meter stick.

The jet observer in  $K_1$  sees the K (lab) meter stick as much shorter than the k (disk) meter stick.

The disk observer in k sees the  $K_1$  (jet) meter stick as shorter than the K (lab) stick.

## 7 Summary and Conclusions

From the logic presented herein, it appears that the traditional approach prediction of Lorentz contraction on the rim of a rotating disk is inconsistent. The NTO approach prediction of no Lorentz contraction appears consistent. These conclusions apply to both the limit and non-limit cases.

The rotating frame limit case has been demonstrated distinguishable from a Lorentz frame due to substantial and measurable decrease in mass of known particle types, which results from the high value for the potential at large radius.

General motion of particles may be described using traditional local Lorentz frame methodology in conjunction with NTO analysis. For NTO analysis, the instantaneous center of rotation is determined from the angular velocity  $\omega$  and the particle acceleration  $\mathbf{a}$ . The degree of non-time-orthogonality is then found from  $\omega r$ .

The effectively physical local metric (associated with coordinates whose values are those one would measure via experiment with physical instruments, i.e., standard meter sticks and clocks) has a general form with unit values for diagonal components and non-zero off diagonal spacetime components. In the special (but far more commonplace) case when time is orthogonal to space, this metric reduces to the Minkowski (or Lorentz) metric.

## Appendix: Transitional Release of a Rotating Frame Meter Stick

Figure 2 depicts the sudden release of a meter stick from a rotating frame as seen by an observer in the lab frame. The graphical construction used is well known and is described briefly in Section 6.2 as well as more extensively in references cited therein.

The release is considered to happen at time  $T = 0$  in the lab frame. The meter stick is shown as observed at a time prior to  $T = 0$  in the portion of the graph below the horizontal (spatial) axis. It is shown again above that as observed just as it is released (endpoint events O and N).

A meter stick traveling in the Lorentz frame having velocity equal to the instantaneous tangent velocity  $\omega r$  would be observed to have end events O and P for an observer in that frame. As the meter stick is released it changes from the rotating frame state to the moving (relative to us, the observers) Lorentz frame state.

The release of the meter stick endpoints comes at events O and N. Note that within the moving Lorentz frame (the “jet” frame of Section 6.2) these events are not simultaneous. N occurs before O therein. Event M occurs shortly after N for the jet frame, but is simultaneous with N for the lab. Using the standard construction from event M, we obtain the representation MQ, where MQ has the same 4D length as MN. Similarly, RS has the same spacetime length as RN; and OP has the same spacetime length as ON. Note that at time  $T_a$ , the length for the meter stick seen in the lab would be VS, and this is part way between non-contracted (as for the disk frame) and Lorentz contracted (as for the jet frame.) The result is a smooth transition of meter stick length as observed in the lab from uncontracted to Lorentz contracted.

## References

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- [2] Einstein, A., and Infeld, L., *The Evolution of Physics* (Simon and Schuster, 1938), pp. 226-234.
- [3] Stachel, J., “Einstein and the Rigidly Rotating Disk”, Chapter 1 in Held, *General Relativity and Gravitation* (Plenum Press, New York, 1980), pp. 1-15.
- [4] T.A. Weber, “Measurements on a rotating frame in relativity, and the Wilson and Wilson experiment”, *Am. J. Phys.* 65 (10), 946-953 October 1997.



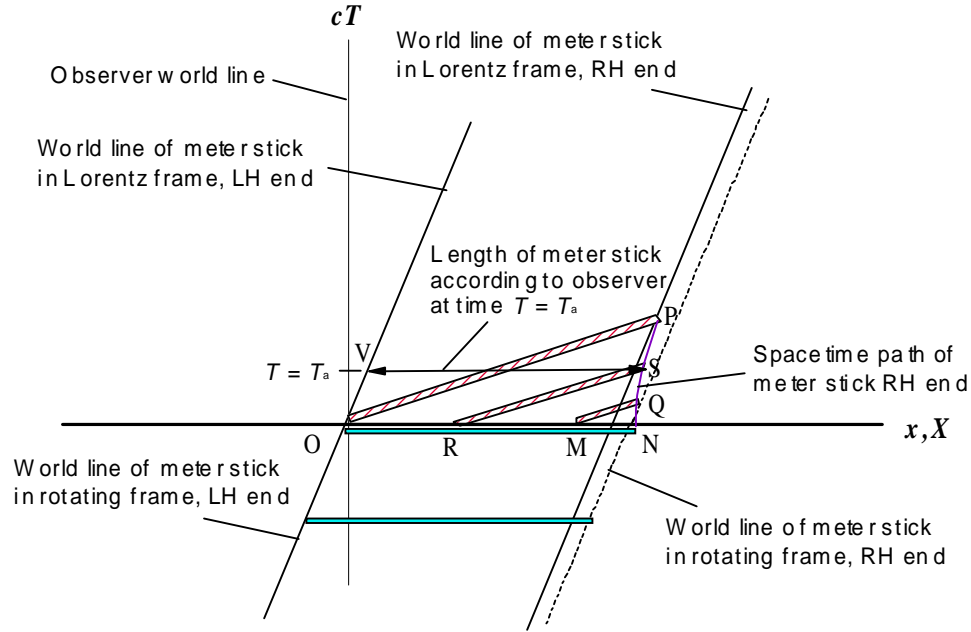


Figure 2: . Meter Stick Suddenly Released from Rotating Frame

- [5] Pathra, *The Theory of Relativity*, eq (6.4), p. 156.
- [6] Richard A. Mould, *Basic Relativity*, Springer-Verlag, N.Y. (1944), p. 272.
- [7] Actually, as speed increases, the disk rim presumably tries to contract, but is prevented from doing so by the structural integrity of the disk material. In this view stress is thus induced circumferentially in the disk purely from relativistic kinematic causes. Increasing speed increases stress and at a certain angular velocity the disk should rupture. However, independent rods or meter sticks laid down around the circumference that are not connected to one another should each Lorentz contract independently, directly, and unfettered. In either case, we shall simplify our discussion by referring to this phenomenon as simply Lorentz contraction of the rim.
- [8] Arzelies, H., *Relativistic Kinematics*, (Pergamon Press, New York 1966), Chapter IX. Arzelies was not convinced that tension arises in the disk, but references others who were.
- [9] Paul Langevin, "Sur la théorie de relativité et l'expérience de M. Sagnac", *Academie des sciences comptes rendus des seances.*, Vol. 173, 831-834 (7 Nov 1921); "Relativité – Sur l'expérience de Sagnac", *Academie des sciences comptes rendus des seances.*, Vol. 205, 304-306 (2 Aug 1937.)
- [10] Robert D. Klauber, "New perspectives on the relatively rotating disk and non-time-orthogonal reference frames", *Found. Phys. Lett.* **11**(5) 405-443 (1998). qr-qc/0103076
- [11] Neil Ashby, "Relativity and the Global Positioning System", *Phys. Today*, May 2002, 41-47. See pg 44; , "Relativistic Effects in the Global Positioning System", *15<sup>th</sup> Intl. Conf. Gen. Rel. and Gravitation*, Pune, India (Dec 15-21, 1997), available at [www.colorado.edu/engineering/GPS/Papers/RelativityinGPS.ps](http://www.colorado.edu/engineering/GPS/Papers/RelativityinGPS.ps). See pp. 5-7. Ashby observes that applying the assumptions implicit in the traditional approach to the rotating earth frame produce errors in the GPS. Use of the NTO approach, however, yields correct results.
- [12] A. Brillet and J. L. Hall, "Improved laser test of the isotropy of space," *Phys. Rev. Lett.*, **42**(9), 549-552 (1979). Brillet and Hall report a persistent signal they designate as "spurious" because it is not predicted by the traditional approach. However, such a signal is predicted by the NTO approach.

- [13] M. G. Sagnac, *Comptes Rendus*, **157**, 708-718 (1913); “Effet Tourbillonnaire Optique. La Circulation de l’é lumineux dans un Interférographe Tournant”, *Journal de Physique Théorique et Appliquée*, Paris, Société française de physique, Series 5, Vol 4 (1914), 177-195. Sagnac states clearly and repeatedly that from the point of view of the rotating apparatus he considers his experiment inexplicable via the traditional approach.
- [14] Alexandre Dufour et Fernand Prunier, “Sur l’observation du phénomène de Sagnac avec une source éclairante non entraînée”, *Academie des sciences comptes rendus des seances.*, Vol. 204, 1322-1324 (3 May 1937); “Sur un Deplacement de Franges Enregistre sur une Plate-forme en Rotation Uniforme”, *Le Journal de Physique et Le Radium*, serie VIII, T. III, No 9, 153-161 (Sept 1942). Dufour and Prunier repeat Sagnac’s experiment under a variety of conditions. They too are emphatic in their belief that the results can not be predicted via the traditional approach.
- [15] Robert D. Klauber, “Derivation of the General Case Sagnac Experimental Result from the Rotating Frame”, gr-qc/0206033. The Sagnac experimental result is derived using the NTO approach. It is submitted that this result has never been derived from the rotating frame using the traditional approach, and that it does not appear possible to do so. Langevin (ref [9]) drew similar conclusions.
- [16] Ref [10] summarizes the similarities of, and differences between, the predictions of the traditional and NTO approaches.
- [17] Tartaglia, A., “Lengths on Rotating Platforms”, *Founds. Phys. Lett.*, **12**(1), 17-28 (1999). Tartaglia and I have made the point in this paragraph independently.
- [18] NTO analysis resolves this issue by limiting the validity of the surrogate local Lorentz frames approach to (non-inertial or inertial) frames in which time is orthogonal to space. While for NTO frames this heretofore sacrosanct principle appears invalid, it continues to be correct for the vast majority of all cases treated in general relativity.
- [19] For example, due to time dilation, the angular velocity  $\omega_k$  measured inside the rotating frame k would be greater than the angular velocity  $\omega$  as seen from the non-rotating frame. In particular,  $\omega_k = \omega / \sqrt{1 - \omega^2 r^2 / c^2}$ . Since relativistically, it can be shown that  $a_k = \omega_k^2 r$ , one can solve these two equations for  $r$  and  $\omega$ , the values typically employed in NTO analysis.
- [20] NTO theory is based on differential geometry, just as is traditional time orthogonal general relativity. The same result is obtained in both. See ref. [10], section 4.3.4, eq (26).
- [21] One might consider that the change in mass measured might be due to a gravitational potential rather than a rotational potential, and thus the system would not be subject to the idiosyncracies of NTO systems. That is, the decrease in mass would not have to be due to a rotational potential *per se*. However, in a gravitational potential *both* observers are immersed in the gravitational field. Hence, each would measure the same mass change for a given particle type at rest with respect to her. For rotation, only the disk based observer would measure such a change. Looking at a mass measurement experiment in the other’s frame and comparing with the same experiment in one’s own frame would readily determine which observer is truly in a rotating frame.
- [22] Again, we are illustrating a principle using relations from non-relativistic mechanics. The same principle is readily shown to be true relativistically.
- [23] Max Born, *Einstein’s Theory of Relativity*, Dover, NY, 238-255 (1965).